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# Lateral Response of Nonlinear Nose-Wheel Landing Gear Models with Torsional Free Play

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**Nose-wheel landing gears can, under certain conditions, exhibit instability in lateral dynamics, causing divergent coupled lateral flexural and torsional oscillations called shimmy. The stability of the system depends on the dynamic characteristics of the gear and tires, nonlinearities in the system, and vibratory modes of the vehicle as a whole, as well as the degree of coupling that exists between them. Shimmy may be caused by a number of conditions such as low torsional stiffness of the strut, free play in the gear, wheel imbalance, or worn parts. Free play in the steering degree of freedom has the potential to significantly reduce the divergent shimmy velocity. Nonlinear behavior of landing gear makes the evaluation of the shimmy phenomenon more complex and its prediction more difficult. This work presents a study of the shimmy instability of a three-degree-of-freedom simplified nose-wheel landing gear model with linear flexible tire model and nonlinearities arising out of torsional free play. Response results are presented for a typical range of values of various problem parameters. Numerical studies bring out several interesting features of the shimmy and its dependence on free play.**

## Nomenclature

$C_S$ = equivalent lateral damping coefficient of the nose-wheel landing gear strut
$C_{\mathrm{Sh}}$ = additional viscous damping (shimmy damping) in the torsional degree of freedom
$C_\Delta$ = lateral damping of the tire
$C_\theta$ = equivalent structural damping coefficient in the torsional degree of freedom
$F_N$ = side force due to lateral flexibility of the tire
$K_S$ = lateral stiffness of the nose-wheel landing gear
$K_\Delta$ = lateral stiffness of the tire
$K_\theta$ = torsional stiffness of the nose-wheel landing gear
$I$ = moment of inertia of wheel-strut assembly about the gear vertical axis
$L$ = distance of axis of wheel rotation from the gear vertical axis
$L_{\mathrm{cg}}$ = distance of the center of gravity of the wheel-strut assembly from the gear vertical axis
$m$ = mass of the wheel-strut assembly
$t$ = dimensional time
$V$ = landing gear forward velocity
$V_{\mathrm{Cr}}$ = critical shimmy velocity
$V_{\mathrm{ts}}$ = tire contact point velocity
$y$ = lateral displacement of the strut
$\Delta$ = lateral displacement of the tire
$\theta$ = rotation of the wheel about the gear vertical axis
$\theta_{\mathrm{fp}}$ = free play in wheel the torsional degree of freedom
$\tau$ = nondimensional time
$\Omega$ = ratio of nose-wheel landing gear strut torsional frequency to lateral bending frequency, $\omega_\theta/\omega_s$
$\omega_s$ = uncoupled nose-wheel landing gear strut lateral bending frequency
$\omega_\theta$ = uncoupled nose-wheel landing gear strut torsional frequency

## I. Introduction

NOSE-WHEEL landing gears (NLGs) can, under certain conditions, exhibit instability in lateral dynamics, causing divergent coupled lateral flexural and torsional oscillations called shimmy. The stability of the landing gear system depends on the dynamic characteristics of the gear, tires, and vibratory modes of the vehicle as a whole, as well as the degree of coupling that exists between various modes of these components. Shimmy may be caused by a number of conditions such as low torsional stiffness of the strut, free play in the gear, wheel imbalance, or worn parts. Landing gears that shimmy are unacceptable, and in fact, a severe occurrence of shimmy can damage the landing gear and its attaching structure, resulting in significant damage. At speeds close to critical (shimmy) velocity, small motions may become unstable and grow, and in severe cases, the pilot may not able to take corrective action, leading to the failure of the gear. It is therefore necessary that landing gear designs ensure adequate margins between the taxi speeds and the critical velocity of shimmy under all operating conditions. This has made analysis for the prediction of NLG shimmy, a necessary component in the aircraft landing gear design.

Nonlinear behavior of landing gear makes the evaluation of the shimmy phenomenon more complex and its prediction more difficult. It is necessary to incorporate appropriate models for landing gear nonlinearities to obtain accurate estimates of the critical velocity of shimmy. When there are nonlinearities, the NLG system may exhibit limit-cycle oscillations, which under certain conditions may induce large-amplitude responses. Such large responses are also loosely referred to as shimmy, though the NLG system may still be exhibiting stable limit-cycle oscillations. The common nonlinearities encountered in landing gears are dry friction between sliding surfaces, nonlinear tire stiffness and damping, and free play in the torsional degree of freedom (DOF). The most significant of these nonlinearities is free play, which has the potential to significantly reduce the divergent shimmy velocity.

The effect of free play has been addressed by the present authors in a number of earlier studies [1–5]. These studies show that NLG

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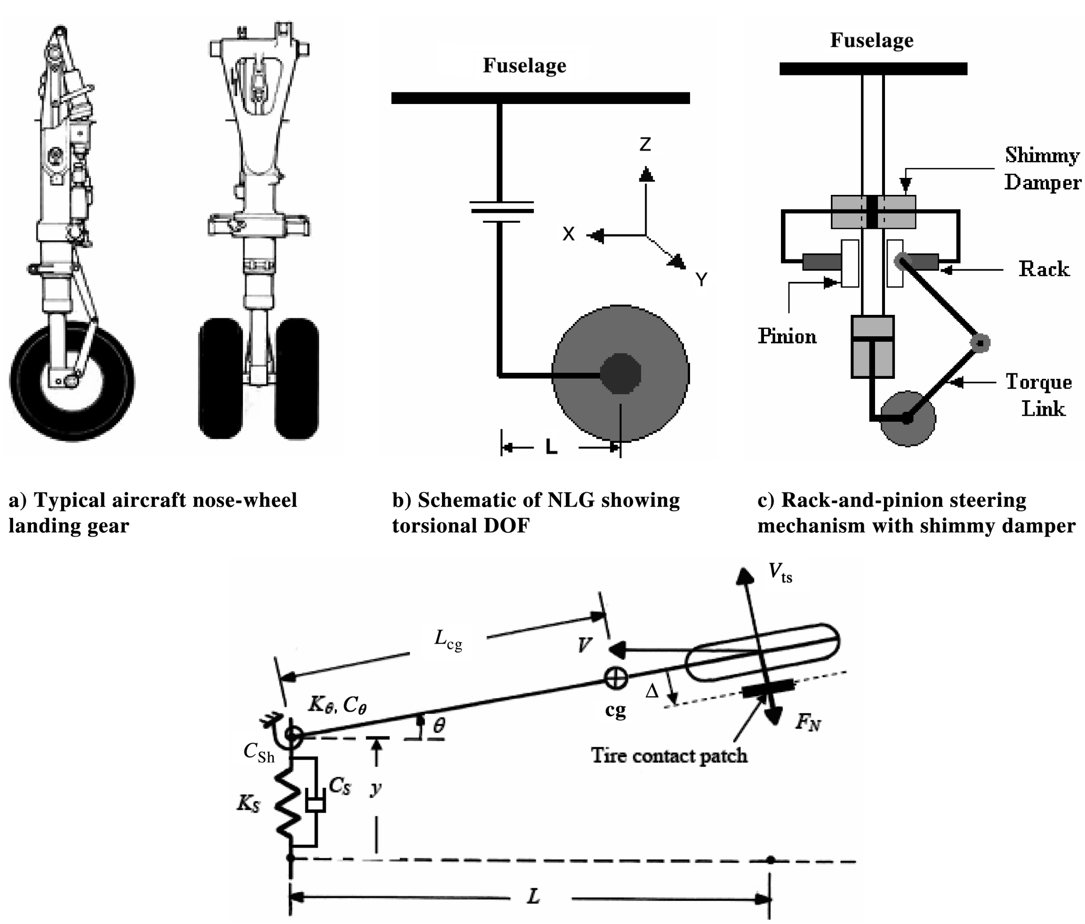
exhibits limit-cycle oscillations even at velocities below divergent shimmy, and the amplitudes of limit-cycle oscillations increase with increase in free play and when the velocity approaches the value of divergent shimmy. Torsional free play tends to destabilize the landing gear, whereas the friction force dissipates energy and thus increases the landing gear stability. Coulomb friction acts opposite to the local relative velocity. However, the torsional friction force generated by the bearings may be reduced to almost zero when the shock absorber stroking velocities are relatively high. The nonlinear flexibility effects of tire and velocity-dependent friction force due to lateral tire slippage with respect to the ground also play an important role on shimmy stability. Further, free play and damping are the only parameters that provide some scope for control in the NLG dynamics at the testing and operational phases of the aircraft. This paper presents a study of shimmy instability of a simplified 3-DOF NLG model with nonlinearities arising out of free play in the torsional DOF.

There have been several studies [6–14] available on modeling and analysis of landing gear nonlinearities, in particular, torsional free play and friction in the system. Nonlinear analysis of landing gear for the study of shimmy has been presented by Krabacher [6,7] using two mathematical models accounting for torsional free play and coulomb friction. Critical nonlinear parametric coefficients in the models have been given by indicating the sensitivity of these parameters to numerical variation. His studies identified free play as one of the stability critical parameters. A nonlinear model of nose landing gear of a typical fighter aircraft has been presented by Baumann [8]. The landing gear model includes inertia, coulomb and viscous damping, stiffness, and torsional free play. Li [9] presented modeling and analysis of NLG shimmy in which the landing gear model includes nonlinearities arising from free play and nonlinear damping in steering system, dry friction between the piston and cylinder, and spring-hardening effects of the bending and torsional stiffness. Gordon Jr. [10] presented an asymptotic method involving the multiple-time-scale perturbation technique for nonlinear stability analysis of NLG shimmy models with velocity-squared damper and obtained general expressions for the limit-cycle amplitude. A perturbation analysis of nonlinear wheel shimmy with coulomb friction and free play has been presented by Gordon Jr. [11]. His studies showed that when only coulomb friction is present, an unstable limit-cycle exists; when only free play is present, a stable limit cycle exists; and when both the nonlinearities are present, both stable and unstable limit cycles exist.

In all the preceding studies [6–11], results have been limited to specific cases of NLG represented by specific values of problem parameters. Because nonlinearities such as free play are highly dependent on wear, it is necessary to examine the potential for occurrence of shimmy for a range of possible values of the system parameters. Limit cycles of simple nonlinear NLG models have been studied by Somieski [12] using various solution techniques. A complete set of nonlinear equations of unsteady tire dynamics of the wheel has been developed by Koenig [13]. It has been demonstrated that reasonable shimmy analyses are possible with these equations. The influence of some of the coefficients responsible for the nonlinearity in the equations has revealed the necessity of adequate tire tests. Woerner and Noel [14] also observed that the effect of free play and friction on shimmy stability is significant.

The problem of analyzing a wheeled system for shimmy also requires the knowledge of various system components, tire characteristics, and how the forces of contact at the road surface are transmitted to the wheel. Valid mathematical relationships describing the force-deflection characteristics of the tire must be used to predict the onset of shimmy correctly. Most studies available on landing gear dynamics models consider either Moreland's point-




**a) Typical aircraft nose-wheel landing gear**

**b) Schematic of NLG showing torsional DOF**

**c) Rack-and-pinion steering mechanism with shimmy damper**

**d) Three-degree-of-freedom nose-wheel landing gear lateral dynamics model**

**Fig. 1 Schematic representation of typical aircraft nose-wheel landing gear and its three-degree-of-freedom lateral dynamics model**

contact model or von Schlippe's stretched-string model [15–19]. In this study, a lateral-tire-dynamics model based on Moreland's point-contact theory is used.

## II. Analytical Formulation of Simplified NLG Lateral Dynamics

Figure 1a shows a typical NLG configuration [20] and Fig. 1b shows a schematic representation of the same as a cantilever supported from the fuselage. The flexibility of the cantilever landing gear may cause fore and aft motion in the $x$ direction and lateral motion in the $y$ direction. The vertical motion in the $z$ direction is absorbed by the oleopneumatic shock absorber. Landing gear may also rotate about the fore and aft axis ($x$ axis) because of lateral bending. The wheel is free to swivel about the vertical axis when the steering is not engaged. This DOF helps with steering the aircraft. The steering moment is transferred to the wheels using a rack-and-pinion mechanism through a torque link. The steering is assumed to be hydraulically controlled and incorporated with two spring-loaded hydraulic steering cylinders that serve as a steering mechanism and are also used as a shimmy damper to subdue torsional oscillations automatically. Damping is accomplished in the shimmy damper by the metering of hydraulic fluid through a small orifice between two cylinders. In the present study, the NLG is assumed to be equipped with such a damping mechanism. A schematic of such a steering mechanism is shown in Fig. 1c.

The NLG model considered for the present paper accounts for structural inertia, stiffness, and structural damping. The torsional stiffness of the strut is the resultant of stiffness offered by torque link, hydraulic spring in the steering actuator, and the structure above the torque link, including the retraction jack. The damping in the system is a contribution of the inherent structural damping in lateral and torsional DOF ($C_S$ and $C_\theta$) and external viscous damping ($C_{\mathrm{Sh}}$) in torsional DOF coming from the steering damping mechanism. The net damping produced by the shimmy damper is taken into account here as an effective viscous damping in parallel with the overall spring stiffness. In the aircraft taxiing phase, cyclic loading conditions encountered by the landing gear because of the runway and other excitations may lead to the wear and tear in some of the mechanical components of the landing gear system. These include mechanical plays (free play) in the rack and pinion of the steering system, interlinkages of the torque link, and fuselage attachment joints and lateral plays in the steering collar and wheel axle. In this study, free play in the steering DOF is considered for the purpose of modeling the nonlinear torsional stiffness of the NLG strut. Other landing gear nonlinearities such as nonlinear lateral strut flexibility, friction in the oleostrut, and nonlinear tire behavior are ignored.

Consider an NLG model with 3-DOF, as shown in Fig. 1d, which has a rigid swiveling member of length $L$ attached to a nonswiveling structure. Let $m$ be the mass of the wheel-strut assembly, let $I$ be the moment of inertia of the wheel-strut assembly about the gear vertical axis, and let $L_{\mathrm{cg}}$ be the distance of the center of gravity of the wheel-strut assembly from the gear vertical axis. The dynamics of the NLG system is formulated in terms of equilibrium along its lateral degrees of freedom (i.e., lateral motion $y$ of the wheel assembly), swivel rotation (torsional DOF) of the wheel about the vertical axis $\theta$, and lateral deflection of the tire contact patch with respect to the wheel center plane $\Delta$. Considering free play in torsion and assuming $\theta$ to be small ($\cos\theta = 1$ and $\sin\theta = \theta$), the equations of motion for the 3-DOF NLG system [2,4] can be written as

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + mL_{\mathrm{cg}}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + C_S\frac{\mathrm{d}y}{\mathrm{d}t} + K_S y + F_N = 0 \qquad (1)$$

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + mL_{\mathrm{cg}}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + (C_\theta + C_{\mathrm{Sh}})\frac{\mathrm{d}\theta}{\mathrm{d}t} + M_\theta + F_N L = 0 \qquad (2)$$

where $K_S$ and $C_S$, respectively, are stiffness and equivalent structural damping of the NLG strut in the lateral direction; $C_\theta$ and $C_{\mathrm{Sh}}$ are, respectively, equivalent structural damping and additional viscous damping (shimmy damping) in the torsional degree of freedom; and $M_\theta$ is the torsional moment that can be expressed in terms of strut torsional stiffness $K_\theta$, wheel swivel rotation $\theta$, and free play in wheel swivel DOF $\theta_{\mathrm{fp}}$ as

$$\begin{aligned} M_\theta &= 0 & -\theta_{\mathrm{fp}} < \theta < \theta_{\mathrm{fp}} \\ M_\theta &= K_\theta(\theta - \theta_{\mathrm{fp}}) & \theta > \theta_{\mathrm{fp}} \\ M_\theta &= K_\theta(\theta + \theta_{\mathrm{fp}}) & \theta < -\theta_{\mathrm{fp}} \end{aligned} \qquad (3)$$

In Eqs. (1) and (2), $F_N$ represents the side force generated by the tire lateral deformation. Linear mathematical relationships describing the force-deflection characteristics of the tire (based on Moreland's model) are considered for the dynamic tire model. It is assumed that the tire is laterally flexible and torsionally rigid. Also, the side force $F_N$ acting on the wheel is proportional to the deflection $\Delta$ and to the rate of change of $\Delta$ with time and can be expressed in terms of tire lateral stiffness $K_\Delta$ and damping $C_\Delta$ as

$$F_N = K_\Delta \Delta + C_\Delta \frac{\mathrm{d}\Delta}{\mathrm{d}t} \qquad (4)$$

If the force $F_N$ is strong enough to resist lateral force due to strut lateral dynamics, then there will not be any slippage of tire with respect to the ground. Here, it is assumed that there is no tire slip with respect to the ground. The kinematic condition that ensures no tire slip with respect to the ground (i.e., $V_{\mathrm{ts}} = 0$) is given by

$$V\theta + \frac{\mathrm{d}y}{\mathrm{d}t} + L\frac{\mathrm{d}\theta}{\mathrm{d}t} - \frac{\mathrm{d}\Delta}{\mathrm{d}t} = 0 \qquad (5)$$

Equations (1), (2), and (5) along with Eqs. (3) and (4) represent the nonlinear lateral dynamics of 3-DOF NLG with free play in the steering DOF. Defining $\omega_S^2 = (K_S/m)$ and $\omega_\theta^2 = (K_\theta/I)$ and the following nondimensional parameters,

$$\bar{y} = \frac{y}{L}, \quad \bar{\Delta} = \frac{\Delta}{L}, \quad \bar{L} = \frac{L_{\mathrm{cg}}}{L}, \quad \bar{I} = \frac{I}{mL^2}, \quad \tau = t\omega_S$$
$$\Omega = \frac{\omega_\theta}{\omega_S}, \quad \bar{V} = \frac{V}{\omega_S L} \quad \bar{K}_\Delta = \frac{K_\Delta}{m\omega_S^2}, \quad \bar{C}_\Delta = \frac{C_\Delta}{m\omega_S}$$
$$\bar{C}_S = \frac{C_S}{m\omega_S}, \quad \bar{C}_\theta = \frac{C_\theta}{mL^2\omega_S}, \quad \bar{C}_{\mathrm{Sh}} = \frac{C_{\mathrm{Sh}}}{mL^2\omega_S}, \quad \bar{M}_\theta = \frac{M_\theta}{mL^2\omega_S^2} \qquad (6)$$

Equations (1), (2), and (5) can be written in nondimensional form as

$$\ddot{\bar{y}} + \bar{L}\,\ddot{\theta} + \bar{C}_S\dot{\bar{y}} + \bar{y} + \bar{K}_\Delta\bar{\Delta} + \bar{C}_\Delta\dot{\bar{\Delta}} = 0 \qquad (7)$$

$$\bar{L}\,\ddot{\bar{y}} + \bar{I}\,\ddot{\theta} + (\bar{C}_\theta + \bar{C}_{\mathrm{Sh}})\dot{\theta} + \bar{M}_\theta + \bar{K}_\Delta\bar{\Delta} + \bar{C}_\Delta\dot{\bar{\Delta}} = 0 \qquad (8)$$

$$\dot{\bar{y}} + \dot{\theta} + \bar{V}\theta - \dot{\bar{\Delta}} = 0 \qquad (9)$$

Equations (7) and (8) represent nonlinear lateral dynamics of 3-DOF NLG with free play in the steering DOF in a nondimensional form. In the preceding equations, single and double dots over the quantities represent first and second derivatives with respect to the nondimensional time parameter $\tau$.

## III. Numerical Results and Discussion

Solution of the complex nonlinear NLG system dynamics requires numerical integration of the system equations using suitable algorithms. It can be seen from the literature [21–24] that for the dynamics of nonlinear structural systems, the Newmark-$\beta$ and the Runge–Kutta methods are the most widely used. In this study, the Newmark-$\beta$ integration scheme is employed to integrate the preceding nonlinear system equations. By solving the incremental equilibrium equation using the local tangent stiffness (treating the nonlinear term as a pseudoforce), solution of the system nonlinear equations for NLG dynamics is obtained. A comprehensive

**Table 1 Values of NLG parameters for 3-DOF baseline problem and their ranges**

| | |
|---|---|
| *Baseline values of NLG parameters* | |
| Strut inertia parameters | $m = 22$ kg and $I_{cg} = 0.198$ kgm$^2$ |
| Strut geometric parameters | $L = 0.075$ m, $L_{cg} = 0.0675$ m, and $\theta_{fp} = 0$ |
| Strut stiffness parameters | $K_S = 542.83$ kN/m and $\Omega = 3$ |
| Strut damping parameters | $\bar{C}_S = 0.01$, $\bar{C}_\theta = 0.02$, and $C_{Sh} = 50$ Nms/rad |
| Tire parameters | $K_\Delta = 238.75$ kN/m and $C_\Delta = 205$ Ns/m |
| *Range of values of NLG parameters* | |
| Forward velocity | $V = 10$–$350$ kmph |
| Shimmy damping | $C_{Sh} = 50$–$200$ Nms/rad |
| Torsional free play | $\theta_{fp} = \pm 5 \times 10^{-4}$– $\pm 3 \times 10^{-2}$ rad |

**Table 2 Damping and frequency at various values of velocity $V$ obtained for different time steps d$\tau$**

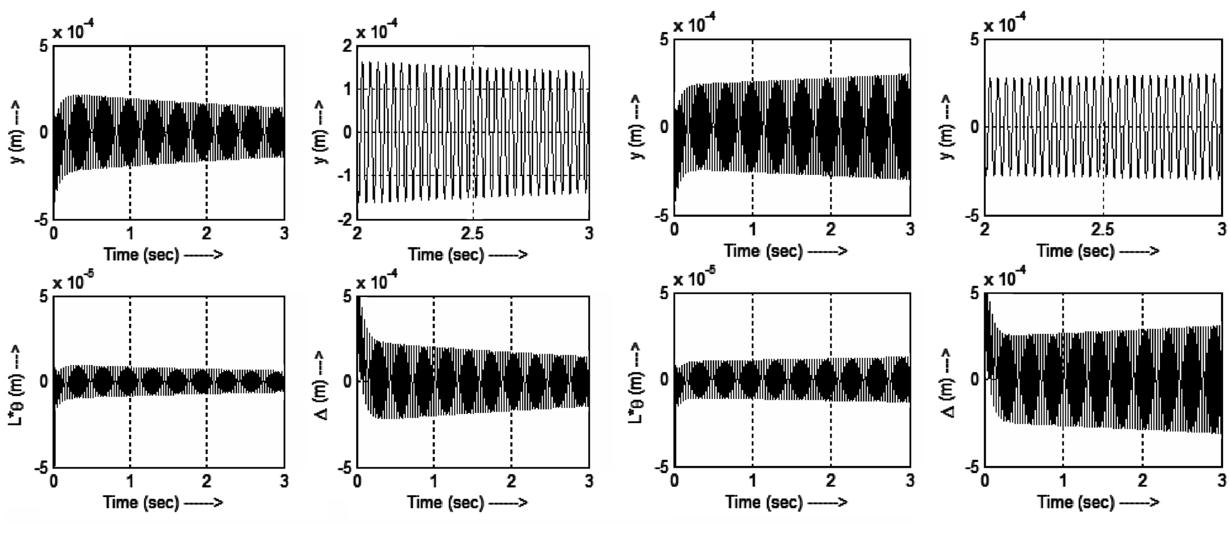
| | Newmark-$\beta$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | d$\tau = 0.001$ | | d$\tau = 0.01$ | | d$\tau = 0.1$ | | Analytical Solution | |
| Velocity $V$, kmph | Damping | Freq, Hz | Damping | Freq, Hz | Damping | Freq, Hz | Damping | Freq, Hz |
| 110 | −0.057 | 17.82 | −0.059 | 17.81 | −0.063 | 17.81 | −0.050 | 17.82 |
| 120 | −0.042 | 18.24 | −0.044 | 18.24 | −0.050 | 18.22 | −0.041 | 18.24 |
| 125 | −0.031 | 18.66 | −0.033 | 18.66 | −0.037 | 18.64 | −0.031 | 18.66 |
| 135 | −0.023 | 19.08 | −0.023 | 19.08 | −0.025 | 19.06 | −0.022 | 19.08 |
| 145 | −0.012 | 19.51 | −0.012 | 19.51 | −0.014 | 19.50 | −0.012 | 19.51 |
| 155 | −0.003 | 19.92 | −0.003 | 19.92 | −0.004 | 19.90 | −0.003 | 19.92 |
| 165 | 0.008 | 20.34 | 0.008 | 20.34 | 0.009 | 20.33 | 0.008 | 20.34 |
| 175 | 0.020 | 20.75 | 0.020 | 20.75 | 0.020 | 20.74 | 0.019 | 20.75 |
| 185 | 0.031 | 21.16 | 0.031 | 21.16 | 0.032 | 21.15 | 0.030 | 21.16 |

description of this incremental equilibrium approach is available in [21].

Starting with the initial condition $\dot{\bar{y}} = 0.02$ and $\dot{\theta} = -0.02$ and setting all other displacements and their time derivatives equal to zero, the equations are integrated and time responses of linear system and nonlinear system with torsional free play are obtained for self-excited oscillations in terms of $y$, $\theta$, and $\Delta$ for various forward-velocity values. By observing the linear and nonlinear system responses, the effect of torsional free play on the NLG lateral response is studied for the baseline and range-of-values problem parameters given in Table 1.

The results presented in this paper correspond to a time step d$\tau = 0.01$, which corresponds to d$t < 6.5 \times 10^{-5}$ s. Table 2 presents the results for damping (calculated based on the logarithmic decrement of the response amplitude) and frequency values using different integration time steps d$\tau$ (nondimensional) and compared with the values obtained from analytical solution (eigenvalue solution of linear problem) for various values of forward velocity $V$ for the case $\bar{I} = 1.47$, $\bar{L} = 0.9$, $\theta_{fp} = 0$, $\bar{C}_\theta = \bar{C}_{Sh} = \bar{C}_S = 0$, and $\Omega = 1/3$. It can be seen from Table 2 that a time step of d$\tau = 0.01$ provides an accurate estimation.

For the linear system without torsional free play (base line configuration), Fig. 2 shows time responses in terms of $y$, $\theta$, and $\Delta$ before (260 kmph) and after (270 kmph) the onset of instability (shimmy). It is observed that the response is predominantly at the coupled lateral-strut frequency for all velocities and the predominant response is lateral displacement of the strut. The time responses presented in Fig. 2 suggest the estimation of the effective damping in the system based on the logarithmic decrement of the response amplitude. Figure 3 shows the variation with forward velocity and the effective damping for the linear NLG model without free play, calculated based on the logarithmic decrement of the response




**a) Time responses at $V = 260$kmph** **b) Time responses at $V = 270$kmph**

**Fig. 2 Time responses of $y$, $\theta$, and $\Delta$ for the linear NLG system without free play ($\theta_{fp} = 0$) and with $C_{Sh} = 50$ Nms/rad.**

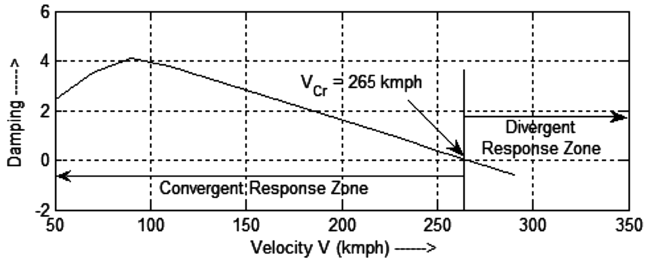
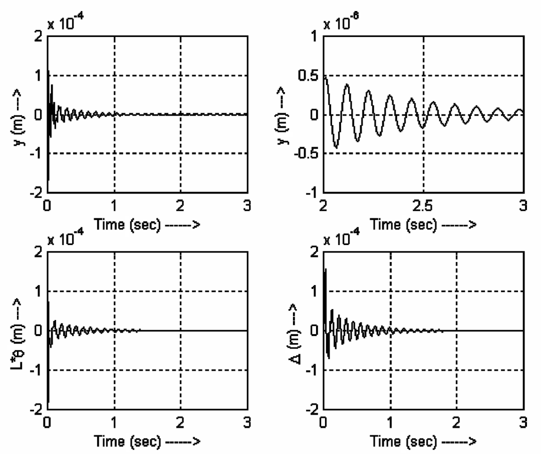


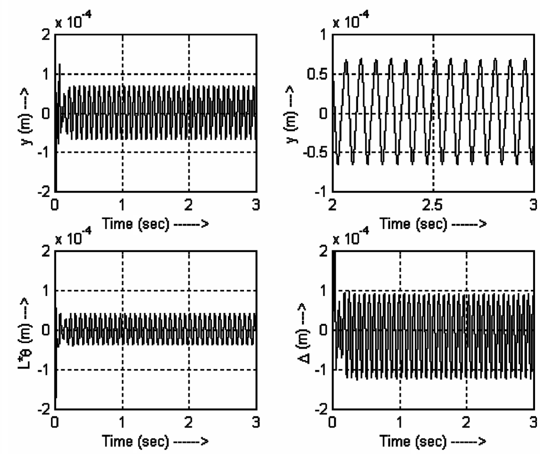

**Fig. 3 Variation of effective damping at various velocities $V$ for the linear NLG system without free play ($\theta_{fp} = 0$) and with $C_{Sh} = 50$ Nms/rad.**

amplitude. It can be seen from Fig. 3 that the onset of instability occurs when damping of the modal response becomes zero at critical velocity $V_{Cr} = 265$ kmph.

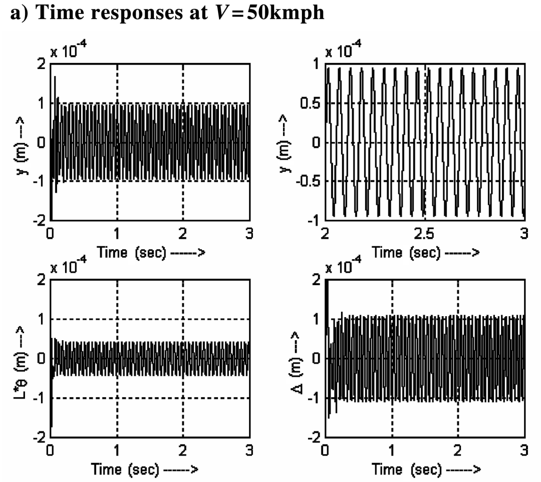
Figures 4a–4d show time responses of the nonlinear system with free play in torsional DOF in terms of $y$, $\theta$, and $\Delta$ at forward-velocity values equal to 50, 100, 150, and 200 kmph for the case $\theta_{fp} = \pm 5 \times 10^{-4}$ rad ($\pm 0.03$ deg), and Fig. 4e shows phase plane plots of lateral response $y$ for the same (preceding) values of velocity $V$. It is observed from Fig. 4 that torsional free play causes limit-cycle oscillations with moderate amplitudes, even in the subcritical velocity range of the corresponding linear system, but causes divergent response beyond a critical velocity, which is much lower



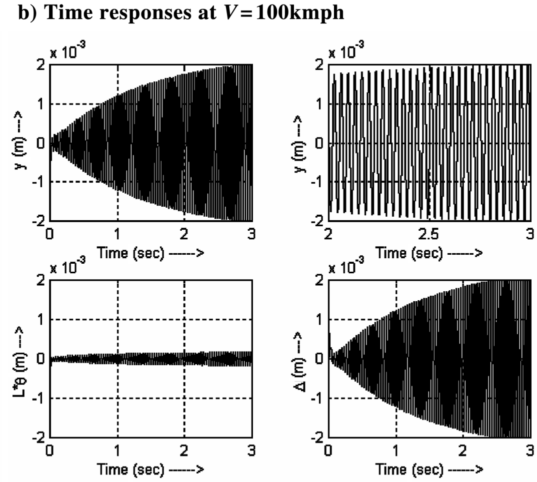
**a) Time responses at $V = 50$kmph**



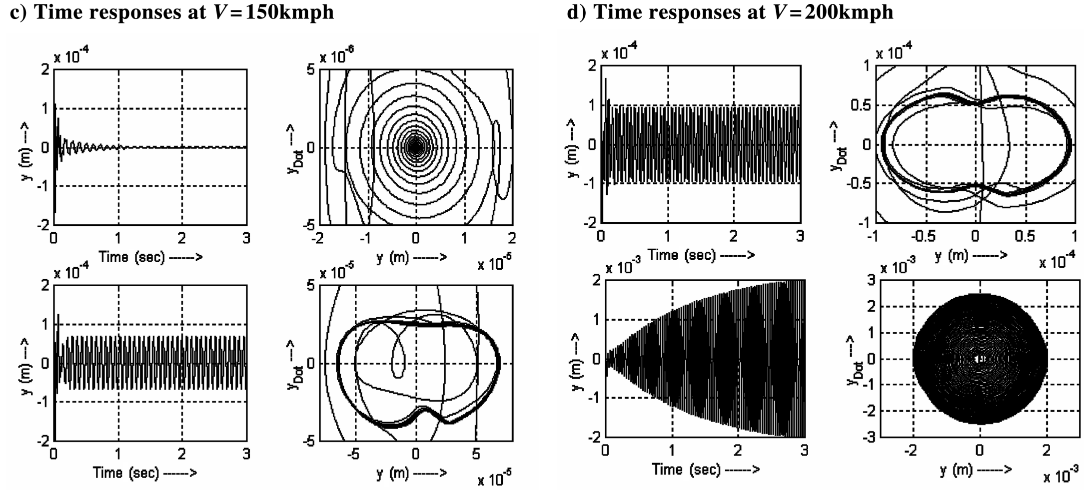
**b) Time responses at $V = 100$kmph**



**c) Time responses at $V = 150$kmph**



**d) Time responses at $V = 200$kmph**



**e) Time responses and phase plane plots of $y$ at $V = 50$, 100, 150, 200kmph**

**Fig. 4 Time responses of $y$, $\theta$, and $\Delta$ at various values of $V$ for the case $\theta_{fp} = 5 \times 10^{-4}$ rad and $C_{Sh} = 50$ Nms/rad.**

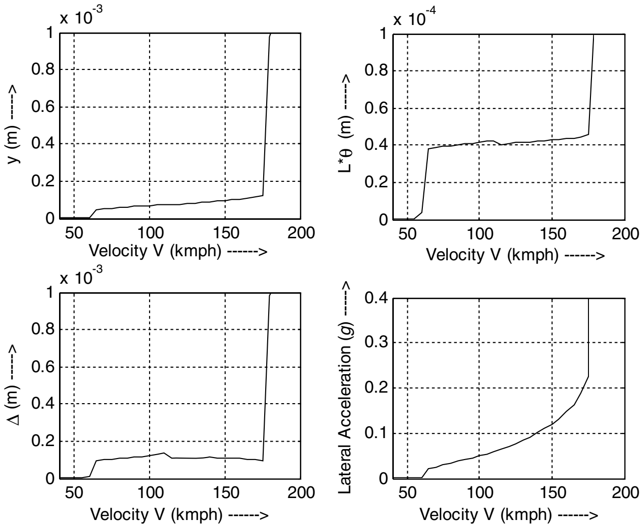



**Fig. 5 Steady-state amplitude values of $y$, $\theta$, and $\Delta$ and lateral acceleration at various velocities $V$ for the system with free play $\theta_{fp} = 5 \times 10^{-4}$ rad and shimmy damping $C_{Sh} = 50$ Nms/rad ($V_{Cr} = 175$ kmph).**

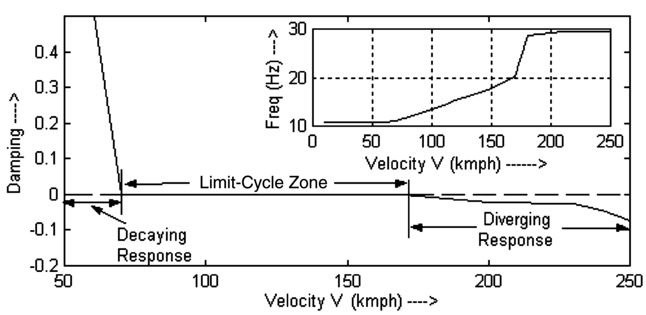



**Fig. 6 Variation of effective damping at various velocities $V$ for the system with free play $\theta_{fp} = 5 \times 10^{-4}$ rad and shimmy damping $C_{Sh} = 50$ Nms/rad.**

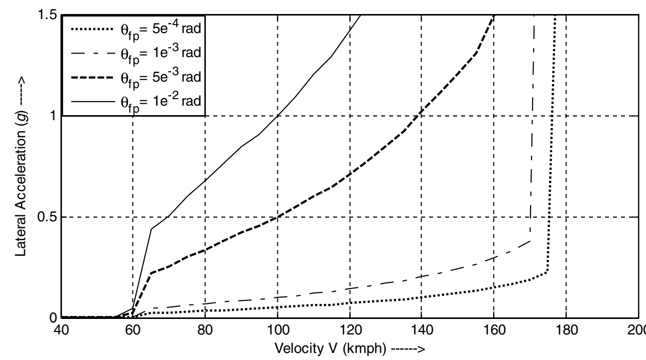



**Fig. 7 Effect of torsional free play $\theta_{fp}$ on NLG strut lateral acceleration: Variation of NLG lateral acceleration response with forward velocity $V$ for various values of $\theta_{fp}$ and for shimmy damping $C_{Sh} = 50$ Nms/rad.**

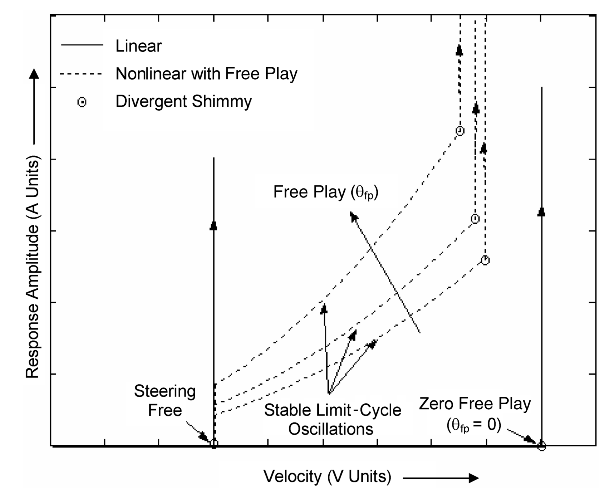



**Fig. 8 Qualitative comparison of NLG lateral acceleration response of linear system without free play with corresponding nonlinear system with free play.**

than the critical velocity of the corresponding linear system. It can be seen that

1) For velocities below a limiting value, the responses decay.

2) For a range of velocities, the response shows steady-state (limit-cycle) oscillations.

3) Beyond a limiting value, the response diverges.

The limit cycles at $V = 100$ and 150 kmph can clearly be seen in Fig. 4e.

Figure 5 shows the variation of response amplitudes for various forward velocities. For the case of the current NLG configuration a free-play value of $\theta_{fp} = \pm 5 \times 10^{-4}$ rad in the steering DOF reduces divergent shimmy to 175 from 265 kmph (the critical velocity of instability corresponding to the linear system without free play). It is seen that below 65 kmph, the response amplitude decays, and above 175 kmph, the response diverges. In the intermediate range of velocity between 65 to 175 kmph, the system exhibits stable limit-cycle oscillations. For the same case, the variation of effective damping based on the logarithmic decrement of the response amplitude and response frequency of the nonlinear NLG with free play is shown in Fig. 6. A clearly identifiable limit-cycle zone is observed over a range of velocities. The lower limit of this zone represents the critical velocity of the system, below which the system is always stable in the presence of free play. This value corresponds to the critical velocity of an NLG system with the steering-free condition ($K_\theta = 0$). This is because when the response amplitude is less than the value of free play, the torsional stiffness of the NLG structure does not come into play. However, the tire lateral stiffness does produce an effective resistance to NLG torsion ($K_\Delta L^2$), and the system displays a lower torsional frequency value equal to 10.68 Hz ($\sqrt{[K_\Delta L^2/4I\pi^2]}$). This can be observed from the plot in Fig. 6, wherein the variation of response frequency with velocity is shown.

The upper limit of the limit-cycle zone gives the value of critical velocity, beyond which the response diverges in the presence of free play. This value depends on the amount of free play. This can be observed in Fig. 7, in which the effect of various values of torsional free play on lateral acceleration response at various velocities is shown. It can also be observed that with increase in torsional free play, peak lateral acceleration levels increases drastically, even in subcritical ranges of velocity. It can be seen from Fig. 7 that increase in the value of free play from $\theta_{fp} = \pm 5 \times 10^{-4}$ to $\pm 5 \times 10^{-3}$ rad increases the peak lateral acceleration levels at the wheel hub from 0.12 to 1.25 $g$ at velocity $V = 150$ kmph.

Figure 8 presents a summary of the preceding results showing a qualitative comparison of the divergent shimmy condition for a linear NLG system and the corresponding nonlinear system with free play. The region of stable limit-cycle oscillations is also shown. It is also seen that the amplitudes of limit-cycle oscillations increase with increase in free play and when the velocity approaches the value of divergent shimmy. Tight control of free play in the steering mechanism can provide scope for keeping these large-amplitude lateral responses at bay. All results presented here for finite free play correspond to the steering-locked condition. When the free play becomes very large, the steering is, in effect, free (with no torsional stiffness). Hence, the divergence velocity for large free play tends to the critical shimmy velocity for a steering-free condition.

Alleviation of large-amplitude responses caused by system nonlinearities can sometimes also be achieved by providing enough

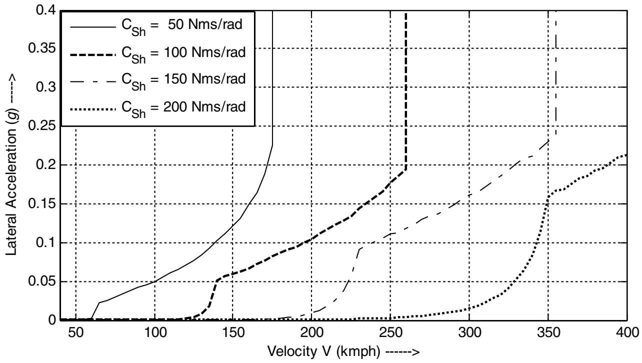



**Fig. 9 Effect of shimmy damping on NLG strut lateral acceleration response: Variation of NLG lateral acceleration response with forward velocity $V$ for various values of shimmy damping $C_{\mathrm{Sh}}$ for the system with free play $\theta_{\mathrm{fp}} = 5 \times 10^{-4}$ rad.**

damping in steering DOF. Figure 9 shows the variation of NLG lateral acceleration response with forward velocity for the case of nonlinear system free play $\theta_{\mathrm{fp}} = 5 \times 10^{-4}$ rad for the shimmy damping $C_{\mathrm{Sh}}$ values 50, 100, 150, and 200 Nms/rad. It can be observed from Fig. 9 that increase in the value of the shimmy damping $C_{\mathrm{Sh}}$ from 50 to 150 Nms/rad decreases the peak lateral acceleration levels at the wheel hub from 0.12 to 0.001 $g$, respectively, at 150 kmph.

## IV. Conclusions

The study of the shimmy instability of a nonlinear NLG model with linear flexible tire model and with torsional free play was presented. Numerical integration based on the Newmark-$\beta$ scheme was used to obtain transient responses of the system. These time-domain simulation studies showed that torsional free play causes limit-cycle oscillations with moderate amplitudes, even in the subcritical velocity range of corresponding linear system, but shows divergent response beyond a critical velocity that is much lower than the critical velocity of the linear system. Even small amounts of free play could cause a significant reduction in the critical velocity. It is observed that there could also be steady-state (stable limit-cycle) oscillations of moderate amplitudes at subcritical velocities that could reduce the margin available for operating velocities.

The preceding observations' impact in practice, as a sudden onset of shimmy instability in aircraft during operation and as wear and tear in the joints and steering mechanism, gradually increases free play in the NLG. In such situations, periodic monitoring and control of free play is the only ready means available to ensure shimmy-free operation.

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